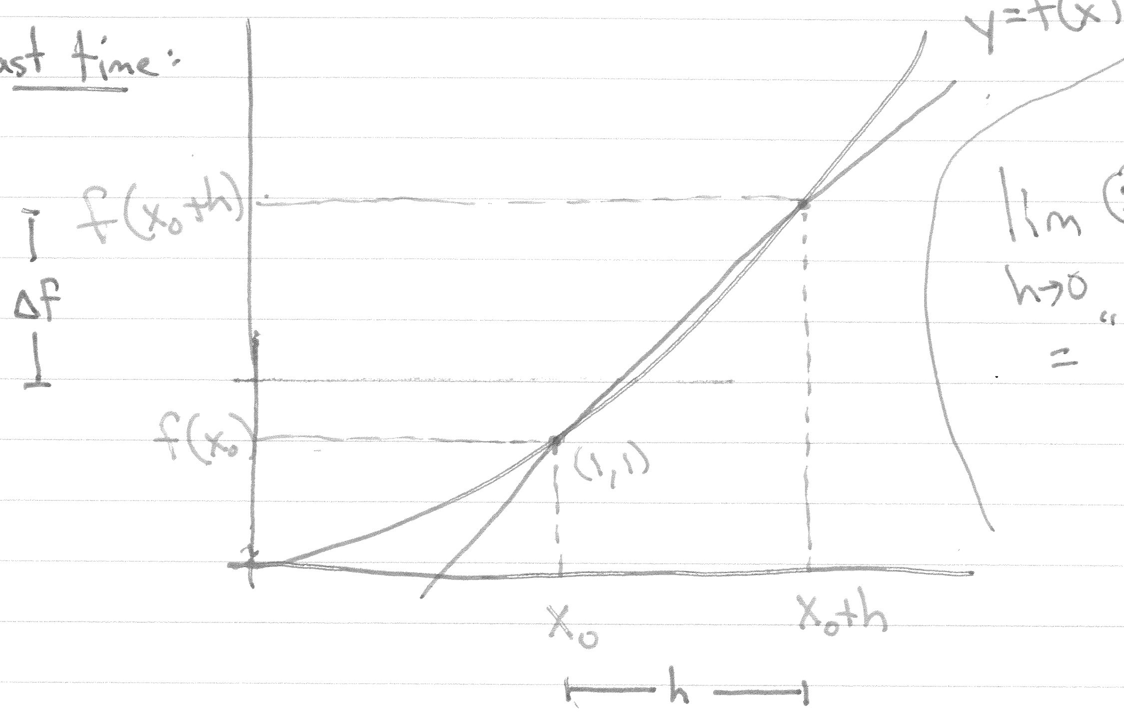
<u>Today</u>:
- Limits, how to compute them
- The derivative
- (If time) Continuity of functions

<u>Last time</u>:



$\lim\limits_{h\to 0} ☺(h)$ = "What does ☺(h) converge to as h goes to 0"

Average rate of change of f over $x_0 \le x \le x_0+h$

$$\frac{\Delta f}{\Delta x} = \frac{f(x_0+h)-f(x_0)}{h} = \text{Slope of secant line.}$$

Idea: Take limit as $h \to 0$ ⇝ <u>tangent line</u>

Slope of the tangent line
||
(Instantaneous rate of change at x_0)

$$= \boxed{\boxed{\lim_{h\to 0}} \frac{f(x_0+h)-f(x_0)}{h}}$$

This is called the derivative of f at x_0

$$f'(x_0) \quad \text{or} \quad \left.\frac{dy}{dx}\right|_{x_0} \quad \text{if } y=f(x)$$

Ex. $f(x)=x^2$, find the derivative at $x_0=1$

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{(1+h)^2-1^2}{h}$$

$$=\lim_{h\to 0}\frac{(1+2h+h^2)-1}{h}$$

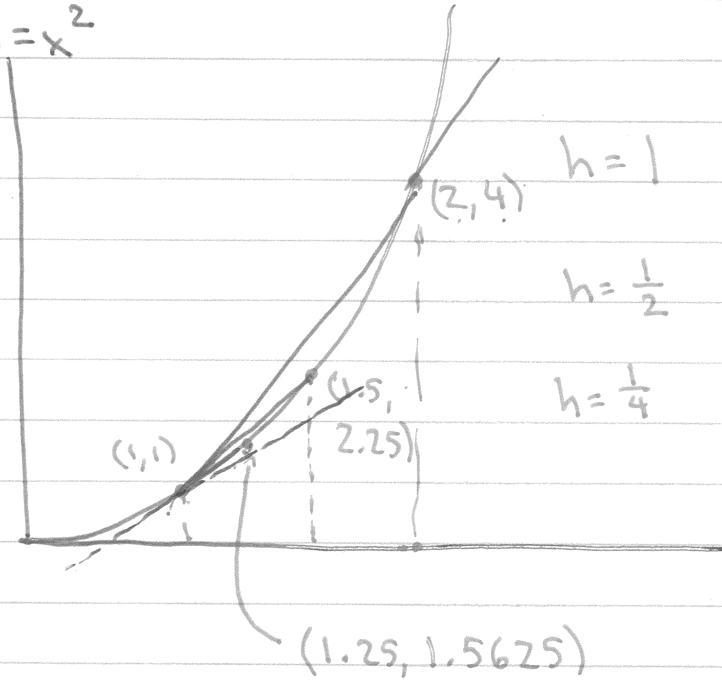
$$=\lim_{h\to 0}\frac{2h+h^2}{h}$$

$$=\lim_{h\to 0}(2+h)=2$$

The slope of the tangent line at $(1,1)$ is 2.

- Approximate using approx parameter h
- Limit as $h\to 0$

Ex. $f(x) = x^2$



$x_0 = 1 \qquad f(x_0) = 1^2 = 1$

$h = 1 \qquad \frac{4-1}{2-1} = 3$

$h = \frac{1}{2} \qquad \frac{2.25-1}{0.5} = \frac{1.25}{0.5} = 2.5$

$h = \frac{1}{4} \qquad \frac{1.5625-1}{0.25} = (.5625)\cdot 4$

$= 2.25$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x \to 2} (x+2) = 4$$

$$\lim_{x \to 2} \frac{x+1}{x-2} \qquad x+1 \to 3, \quad x-2 \to 0$$

$x = 1.9 \qquad \frac{2.9}{-0.1} = -29$

$x = 1.99 \qquad \frac{2.99}{-0.01} = -299$

-2999

$\vdots$

Bonus page!

Examples of computing harder derivatives.

Ex. $f(x)=x^2$

$$f'(x_0)=\lim_{h\to 0}\frac{(x_0+h)^2-x_0^2}{h}=\lim_{h\to 0}\frac{2x_0h+h^2}{h}=\lim_{h\to 0}(2x_0+h)=2x_0.$$

Ex. $f(x)=\frac{2}{3+x}$

$$f'(x_0)=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}=\lim_{h\to 0}\frac{\frac{2}{3+x_0+h}-\frac{2}{3+x_0}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{2(3+x_0)-2(3+x_0+h)}{(3+x_0+h)(3+x_0)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{-2h}{(3+x_0+h)(3+x_0)}}{h}$$

$$=\lim_{h\to 0}\frac{-2}{(3+x_0+h)(3+x_0)}=\frac{-2}{(3+x_0)^2}$$